一、　（10 分）已知系统频率响应$H(j\omega)=\frac{1}{2+j\omega}$，激励$x(t)=e^{-t}u(t)$，求系统零状态响应$y(t)$。

二、　（20 分）已知线性时不变离散时间系统的差分方程为$y[n+2]-5y[n+1]+6y[n]=x[n+1]-x[n]$，激励信号为$x[n]=u[n]$。试求：

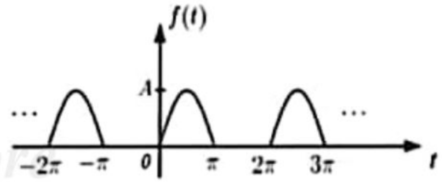
1、$H(z)$，画出直接型模拟方框图；2、单位冲激响应$h[n]$；3、已知$y_{zi}[0]=1, y_{zi}[1]=2$，求系统零输入响应$y_{zi}[n]$；4、求系统零状态响应$y_{zs}[n]$。

三、　（10 分）判断下列信号是能量信号还是功率信号，若是能量信号计算其能量，若是功率信号计算其功率。1. $x_1(t)=e^{-2t}u(t)$，2. $x_2(t)=\cos t$

四、　（10 分）根据奇异函数的性质计算下列各式：（其中t_0是实常数）1. $\int_{-\infty}^{\infty}\delta(t-t_0)u(t-4t_0)dt=$，　2. $\frac{d}{dt}\left(f(t)*\left(u(t)-u(t-t_0)\right)\right)=$

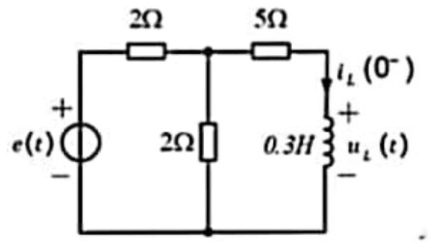
五、　（10 分）已知系统的单位冲激响应$h(t)=e^{-3t}u(t)$，用时域分析法求：

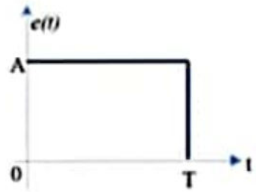
1、激励$x(t)=u(t-3)-u(t-5)$时的零状态响应$y(t)$；2、激励$x_1(t)=\frac{dx(t)}{dt}$时的零状态响应$y_1(t)$。



六、　（20 分）$f(t)$是一个正弦半波整流信号，如图所示，请将它展开为三角傅里叶级数。

七、　（20 分）电路及各元件参数如图所示，电感 L 的初始电流$i_L(0^-)=1A$。





1.　画出运算等效电路；

2.　求电感两端电压的零输入响应$u_{Lzi}(t)$；

3.　以电感两端电压$u_L(t)$作为响应，求系统函数$H(s)$及其单位冲激响应$h(t)$；

4.　已知激励信号$e(t)$的波形如图所示，求电感两端电压的零状态响应$u_{Lzs}(t)$。

一、$x(t) = e^{-t} u(t)$ ∴ $X(j\omega) = \frac{1}{1+j\omega}$

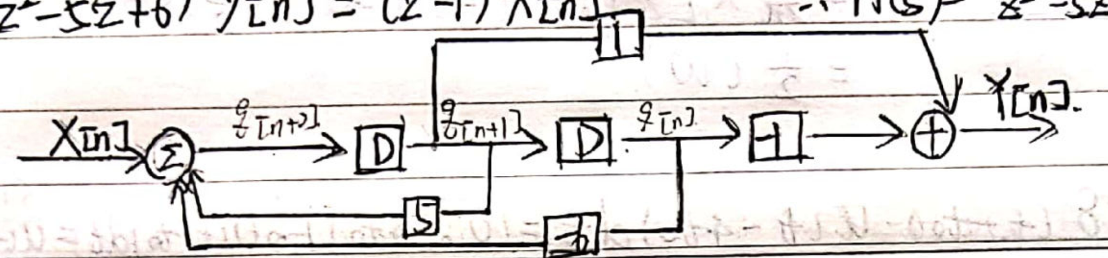
∵ $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{2+j\omega}$

∴ $Y(j\omega) = H(j\omega) \cdot X(j\omega) = \frac{1}{(2+j\omega)(1+j\omega)} = \frac{1}{1+j\omega} - \frac{1}{2+j\omega}$

∴ $y(t) = (e^{-t} - e^{-2t})\, u(t)$.

二、1. $(z^2-5z+6)\, y[n] = (z-1)\, x[n]$ ∴ $H(s) = \frac{z-1}{z^2-5z+6}$

∴



2. $H(z) = \frac{z-1}{(z-2)(z-3)} = \frac{-1}{z-2} + \frac{2}{z-3}$ $\Rightarrow \lambda_1 = 2;\ \lambda_2 = 3$.

∴ 设 $h_1(n) = C_1 \cdot 2^n + C_2 \cdot 3^n$ ∴ $\begin{cases} h_1(0) = \delta(0) + 5h_1(-1) - 6h_1(-2) = 1 \\ h_1(1) = \delta(1) + 5h_1(0) - 6h_1(-1) = 5 \end{cases}$

∴ $\begin{cases} C_1 + C_2 = 1 \\ 2C_1 + 3C_2 = 5 \end{cases} \Rightarrow \begin{cases} C_1 = -2 \\ C_2 = 3 \end{cases}$ ∴ $h_1(n) = 3^{n+1} - 2^{n+1}$.

∴ $h[n] = h_1(n-1) - h_1[n-2] = (3^n - 2^n) - (3^{n-1} - 2^{n-1}) = (2 \cdot 3^{n-1} - 2^{n-1})\, u[n]$.

3. $y[n+2] - 5y[n+1] + 6y[n] = 0$. $y_{zi}[0] = 1$; $y_{zi}[1] = 2$

∴ $z^2 Y_{zi}(z) - z^2 y_{zi}(0) - z y_{zi}(1) - 5z Y_{zi}(z) + 5z y_{zi}(0) + 6Y_{zi}(z) = 0$.

∴ $(z^2 - 5z + 6) Y_{zi}(z) - z^2 - 2z + 5z = 0$.

∴ $Y_{zi}(z) = \frac{z^2-3z}{z^2-5z+6}$ ∴ $\frac{Y_{zi}(z)}{z} = \frac{z-3}{z^2-5z+6} = \frac{1}{z-2}$ ∴ $Y_{zi}(z) = \frac{z}{z-2}$

∴ $y_{zi}[n] = 2^n u[n]$

4. $H(z) = \frac{z-1}{z^2-5z+6}$; $X(z) = \frac{z}{z-1}$

∴ $Y_{zs}(z) = X(z) \cdot H(z) = \frac{z}{z^2-5z+6} = \frac{3}{z-3} - \frac{2}{z-2}$

∴ $y_{zs}[n] = 3 \cdot 3^{n-1} u[n-1] - 2 \cdot 2^{n-1} u[n-1]$

$= 3^n u[n-1] - 2^n u[n-1]$.

廿丁

三、1. 能量信号；$E=\int_{-\infty}^{+\infty}|x(t)|^2dt=\int_0^{+\infty}e^{-4t}dt=-\frac{1}{4}e^{-4t}\Big|_0^{+\infty}$

$=-\frac{1}{4}(0-1)=\frac{1}{4}$ (J).

2. 功率信号：$P=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}|x(t)|^2dt$ ；$T=\frac{2\pi}{1}=2\pi$.

$=\frac{1}{2\pi}\int_{-\pi}^{\pi}\cos^2t\,dt$.

$=\frac{1}{2\pi}\int_{-\pi}^{\pi}(\frac{\cos 2t}{2}+\frac{1}{2})dt$.

$=\frac{1}{2\pi}\times[\frac{1}{4}\sin 2t\Big|_{-\pi}^{\pi}+\frac{1}{2}t\Big|_{-\pi}^{\pi}]$.

$=\frac{1}{2\pi}\times[\frac{1}{4}[\sin 2\pi-\sin(-2\pi)]+\frac{1}{2}\times 2\pi]$.

$=\frac{1}{2}$ (W).

四、1. $\int_{-\infty}^{\infty}\delta(t-t_0)u(t-4t_0)dt=u(t_0-4t_0)\int_{-\infty}^{\infty}\delta(t-t_0)dt=u(-3t_0)=\begin{cases}1, & t_0\le 0\\ 0, & t_0>0\end{cases}$

2. $\frac{d}{dt}[f(t)*(u(t)-u(t-t_0))]$

$f(t)*u(t)=\int_{-\infty}^{t}f(\tau)d\tau$　　$f(t)*u(t-t_0)=f(t-t_0)*u(t)$

$=\int_{-\infty}^{t}f(\tau-t_0)d\tau$.

∴原式 $=f(t)-f(t-t_0)$

五、1. $H(j\omega)=\frac{1}{3+j\omega}$. ∵$\begin{cases}u(t-3)\Leftrightarrow e^{-j\omega\cdot 3}(\pi\delta(\omega)+\frac{1}{j\omega}).\\ u(t-5)\Leftrightarrow e^{-j\omega\cdot 5}(\pi\delta(\omega)+\frac{1}{j\omega}).\end{cases}$

∴$X(j\omega)=(e^{-3j\omega}-e^{-5j\omega})[\pi\delta(\omega)+\frac{1}{j\omega}]$.

∴$Y(j\omega)=H(j\omega)\cdot X(j\omega)=\frac{1}{3+j\omega}(e^{-3j\omega}-e^{-5j\omega})[\pi\delta(\omega)+\frac{1}{j\omega}]$.

$y(t)=\mathcal{F}^{-1}[Y(j\omega)]=$

2. $x_1(t)=\frac{dx(t)}{dt}=\delta(t-3)-\delta(t-5)$　∴$X(j\omega)=e^{-3j\omega}-e^{-5j\omega}$.

∴$Y(j\omega)=X(j\omega)\cdot H(j\omega)=\frac{1}{3+j\omega}(e^{-3j\omega}-e^{-5j\omega})$.

$y_1(t)=\mathcal{F}^{-1}[Y(j\omega)]=$

1.

六、解：$f(\omega t) = \begin{cases} 0 & (2k\pi - \pi \le \omega t \le 2k\pi) \\ A\sin(\omega t) & (2k\pi \le \omega t \le 2k\pi + \pi) \end{cases}$.

$$a_0 = \frac{1}{\pi}\int_0^{\pi} A\sin\omega t\, dt = \frac{A}{\pi}(-\cos\omega t)\Big|_0^{\pi}$$

$$= \frac{A}{\pi}[-(-1)+1] = \frac{2A}{\pi}.$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} A\sin\omega t\cos(n\omega t)\, d\omega t.$$

$$= \frac{A}{\pi}\int_0^{\pi}\frac{1}{2}\{\sin[(n+1)\omega t] + \sin[(1-n)\omega t]\}\, d\omega t$$

$$= \frac{A}{2\pi}\left[\frac{-1}{n+1}\cos(n+1)\omega t + \frac{-1}{1-n}\cos(1-n)\omega t\right]\Big|_0^{\pi}.$$

$$= \frac{A}{2\pi}\left[\frac{2}{1-n^2} + \frac{\cos(n\pi)}{1+n} + \frac{\cos(n\pi)}{1-n}\right].$$

$$= \frac{A}{(1-n^2)\pi}[1+\cos(n\pi)].$$

$$= \begin{cases} \frac{2A}{(1-4k^2)\pi}; & n = 2k \\ 0; & n = 2k+1. \end{cases}$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} A(\sin\omega t)\sin(n\omega t)\, d(\omega t).$$

$$= -\frac{A}{2\pi}\int_0^{\pi}\{\cos(n+1)\omega t] - \cos[(1-n)\omega t]\, d(\omega t).$$

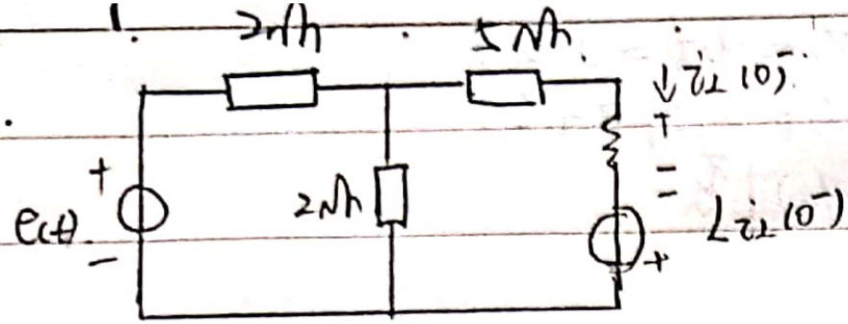
$$= -\frac{A}{2\pi}\left\{\frac{1}{n+1}\sin(n+1)\omega t] + \frac{1}{1-n}\sin[(1-n)\omega t]\right\}\Big|_0^{\pi}.$$

$$= \frac{A}{(1-n)^2\pi}\sin(n\pi).$$

$n=1$ 时洛必达 $\therefore b_1 = \frac{A}{2}$

$$\therefore f(\omega t) = \frac{A}{\pi} + \frac{A}{2}\sin\omega t + \frac{2A}{\pi}\sum_{k=1}^{\infty}\frac{\cos(2k\omega t)}{1-4k^2}.$$

七.1.



2. 解 $u_{Lzi}(t) = L \cdot i_L(0^-) = 0.3 \times 1 = 0.3V.$

3. 解: $R_L = SL = 0.3S\ \Omega$.

$\therefore R_总 = 2 + 2 // [5+0.3S] = \frac{24+1.2S}{7+0.3S}$

设 $R_1 = R_{[2//(5+0.3S)]} = \frac{10+0.6S}{7+0.3S}$

$\therefore H(s) = \frac{R_1}{R_总} \times \frac{0.3S}{5+0.3S} = \frac{1.8S^2+30S}{3.6S^2+132S+1200} = \frac{3S^2+50S}{6S^2+220S+2000}$

令 $6S^2+220S+2000=0$ $S_1=-20$ $S_2=-\frac{50}{3}$

$y(t) = C_1 e^{-20t} + C_2 e^{-\frac{50}{3}t}$ 代入 $\begin{cases} y'(0)=1 \\ y(0)=0 \end{cases}$

$\therefore \begin{cases} -20C_1 - \frac{50}{3}C_2 = 1 \\ C_1 + C_2 = 0 \end{cases}$ $\therefore \begin{cases} C_1 = -\frac{3}{10} \\ C_2 = \frac{3}{10} \end{cases}$

$\therefore h_0(t) =$